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institute of neuroinformatics

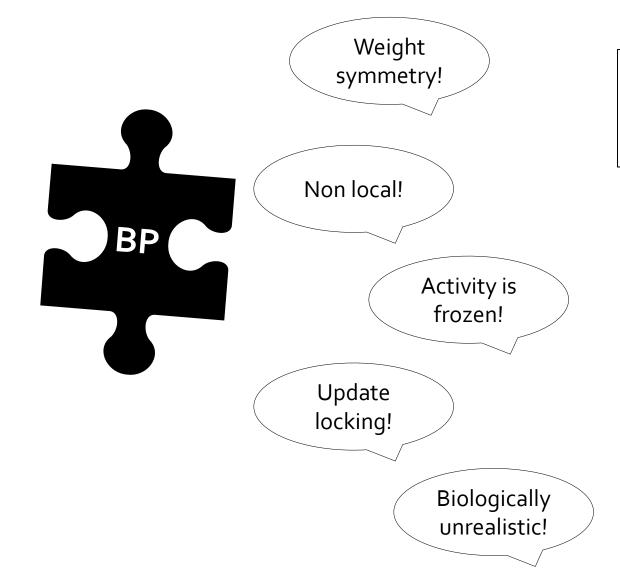
Forward Learning with Top-Down Feedback: Solving the Credit Assignment Problem without a Backward Pass

Dellaferrera & Kreiman, ICML 2022

Srinivasan, Mignacco, Sorbaro, Cooper, Refinetti, Kreiman, <u>Dellaferrera</u>, 2023 (arXiv:2302.05440)



Connecting the puzzle pieces of bio-inspired learning algorithms



Backpropagation and the brain

Timothy P. Lillicrap, Adam Santoro, Luke Marris, Colin J. Akerman and Geoffrey Hinton

Review

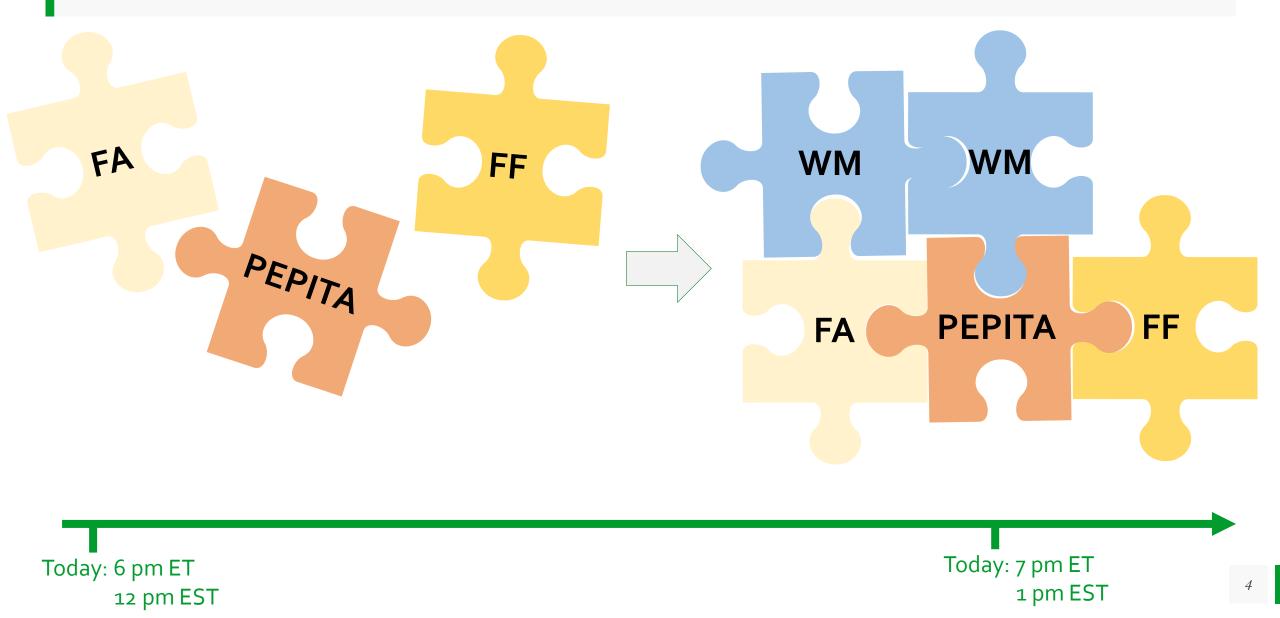
Francis Crick

Theories of Error Back-Propagation in the Brain

James C.R. Whittington^{1,2} and Rafal Bogacz^{1,*}

Connecting the puzzle pieces of bio-inspired learning algorithms 2028 DRTP KP FA FF PEPITA SS WM 2022 IFA 2016 2022

Connecting the puzzle pieces of bio-inspired learning algorithms



Outline

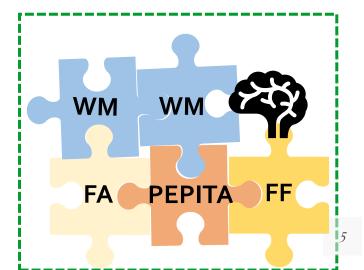
- » Neuro-inspired AI
 - Why Backpropagation is biologically implausible
 - Overview of alternative solutions to credit assignment
- » PEPITA: error-driven input modulation
 - Replacing the backward pass with a second forward pass
 - Results on image classification tasks
 - Soft alignment dynamics
 - Approximating PEPITA to Adaptive Feedback Alignment: analytical characterization
 - Improving alignment with weight mirroring
- » Forward-Forward algorithm
 - Idea and results
 - Similarities with PEPITA's update rule
- » Forward learning with top-down feedback
 - Biological considerations







Followed by **coding tutorial**: Implementing PEPITA with Pytorch



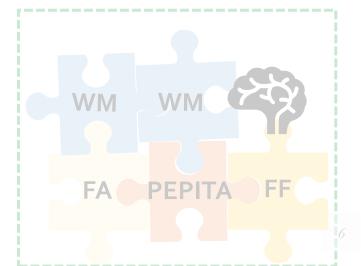
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Backpropagation: successful but not biologically plausible

» Success

- The most effective training algorithm
- State-of-the-art performance in complex cognitive tasks

» Algorithm

- Chain rule of calculus
- Change in synaptic strength \leftrightarrow change of network's error



https://www.bbc.com/news/technology-35785875



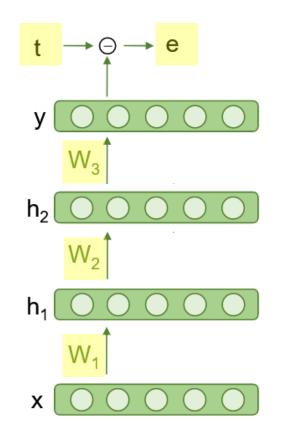




Rumelhart et al., 1989 Crick, 1989 Lillicrap et al., 2020

Glossary

- » Credit assignment
 - Determine the degree to which a parameter (*e.g.*, synaptic weight) has contributed to the network's error
- » Target (t)
 - Desired output of a network
- » Error (e)
 - Deviation of the network's output from the target
- » Weights (W)
 - Parameter corresponding to the strength of the connection between two nodes
- » ANNs = Artificial Neural Networks FCNNs = Fully Connected Neural Networks CNNs = Convolutional Neural Networks



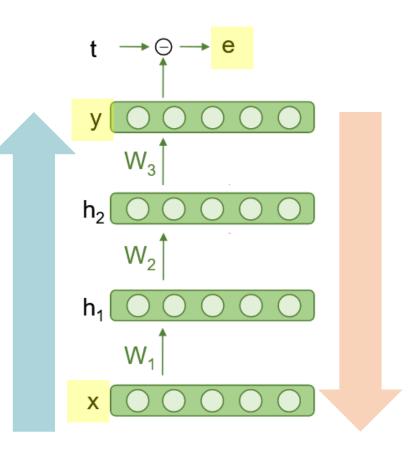
The backpropagation algorithm

» Forward pass

- Network's response to input
- Error function e = y t
- Weight updates proportional to its negative gradient

» Backward pass

- Error signal flows backward through the network
- Computed recursively via the chain rule
- Update phase



Backpropagation: successful but not biologically plausible

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- The most effective training algorithm
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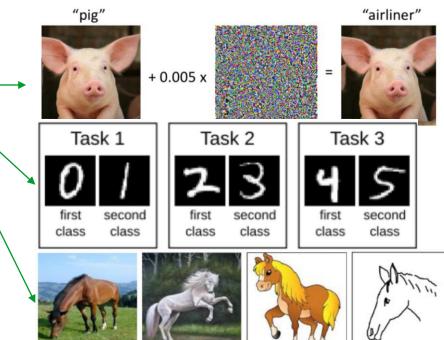
» Algorithm

- Chain rule of calculus
- Change in synaptic strength \leftrightarrow change of network's error

» Criticisms

- Adversarial attack
- Catastrophic forgetting <
- Lack of generalization
- Biological implausibility

Rumelhart et al., 1989 Crick, 1989 Lillicrap et al., 2020

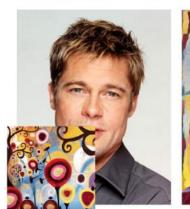




https://www.bbc.com/news/technology-35785875

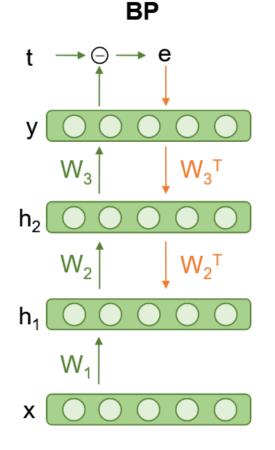








Backpropagation of the error is not biologically plausible



Rumelhart et al., 1995

» Weight transport problem

• Symmetric weights for forward and backward computation

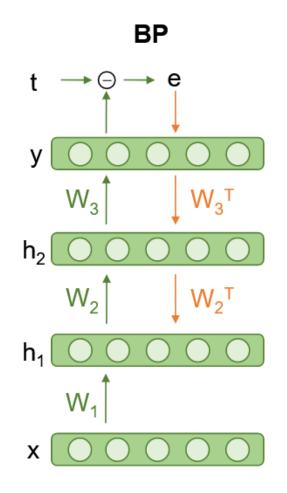
» Non-local information used for the updates

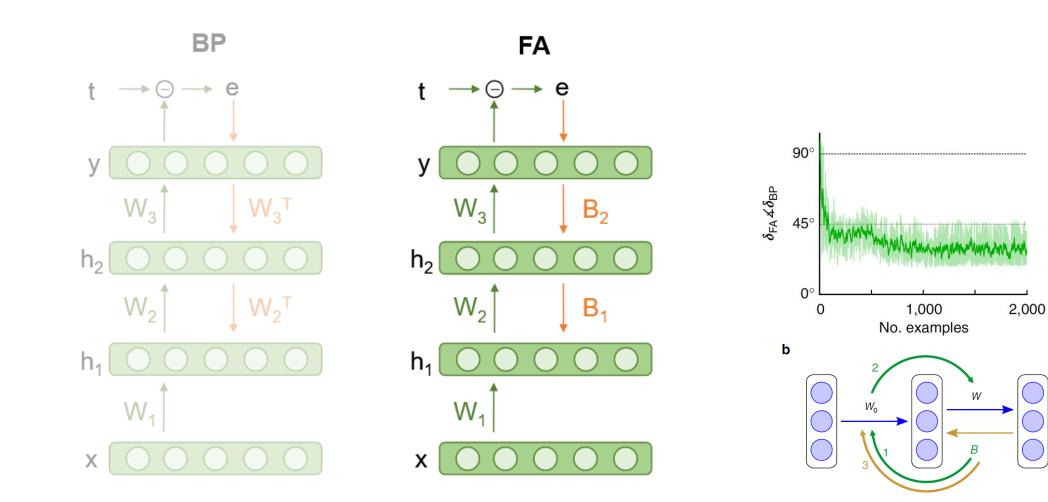
• Global error and downstream weights needed for learning

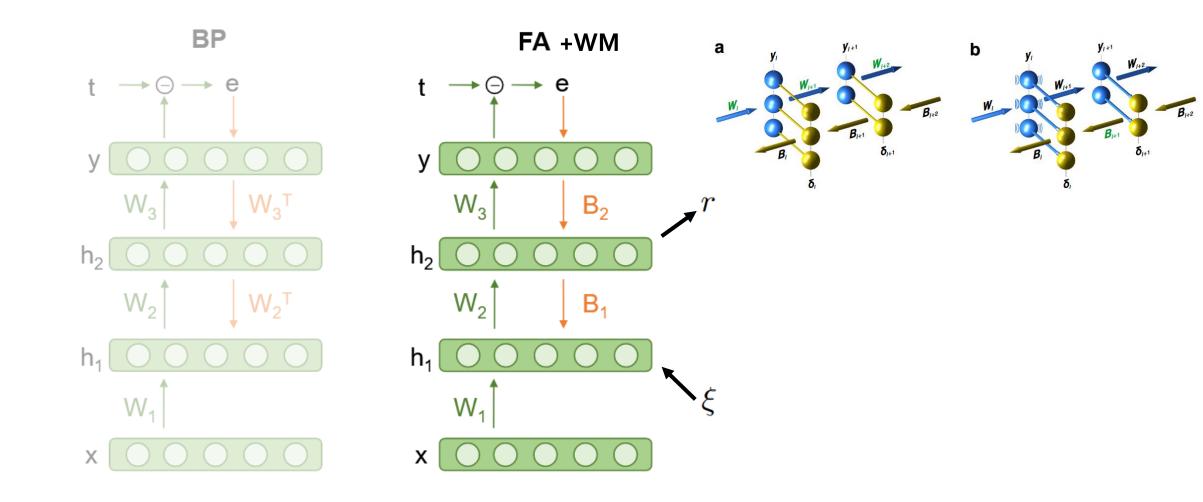
» Frozen activity during error propagation and parameter updates

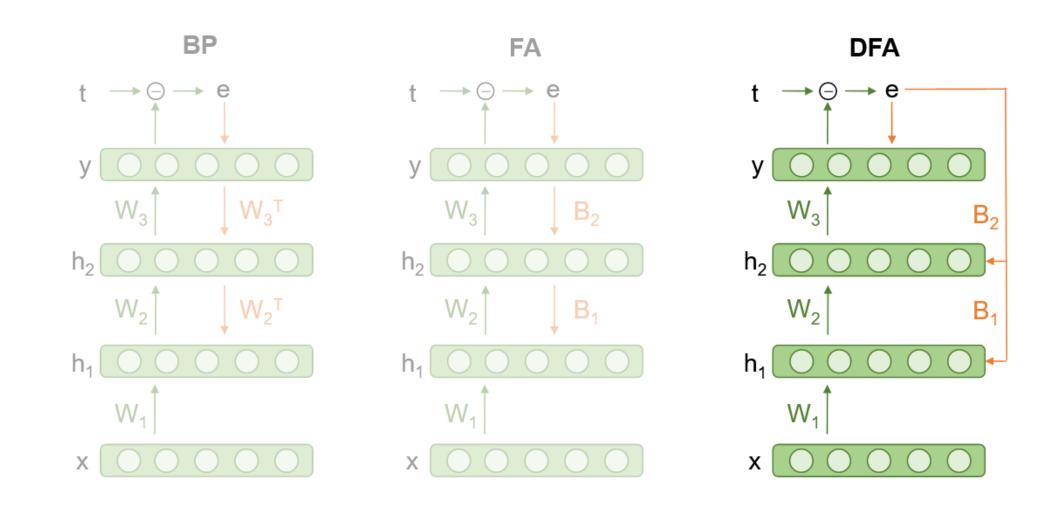
- Separate forward and backward computation
- » Update locking problem
 - Backward computation needs to be complete before the next forward pass

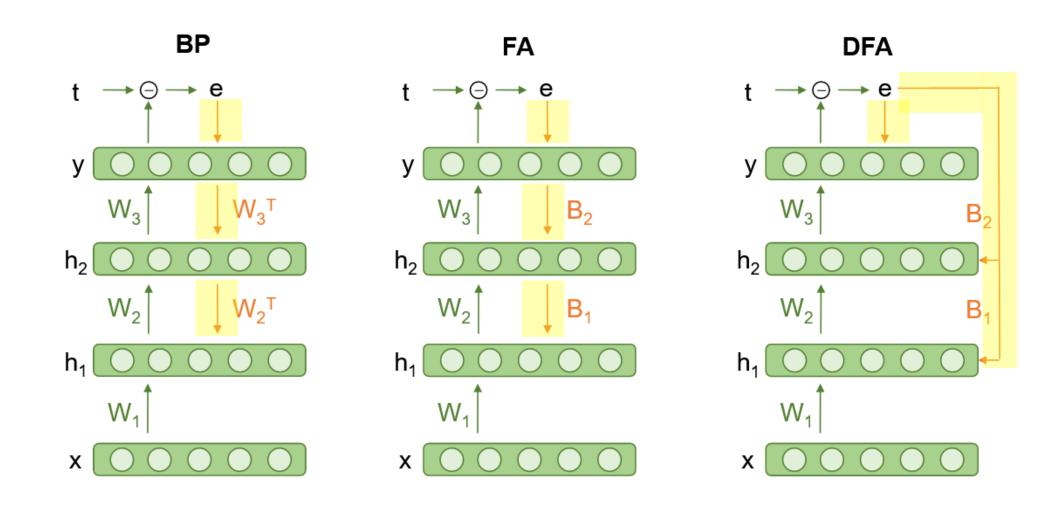
Alternative Training Schemes







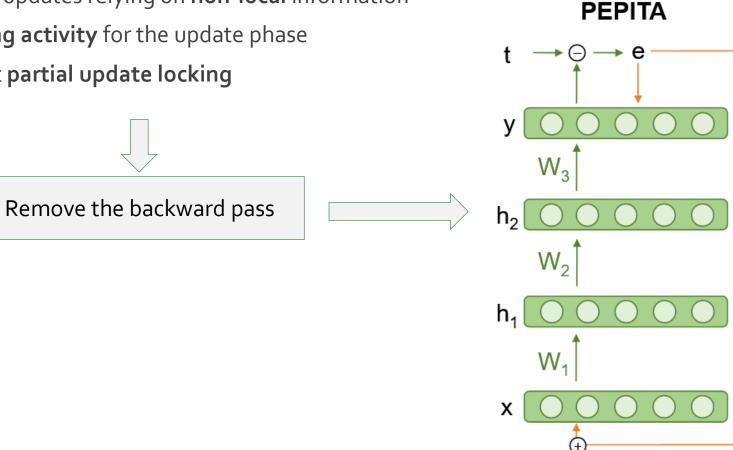




The Backward Pass

The backward pass implies:

- » Weight updates relying on **non-local** information
- » Freezing activity for the update phase
- At least partial update locking **>>**



F

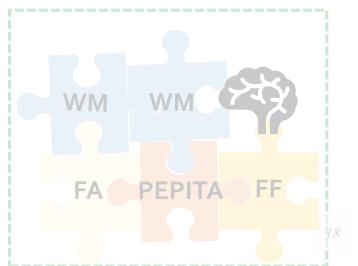
Outline

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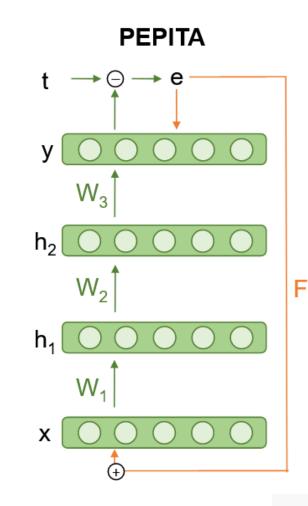






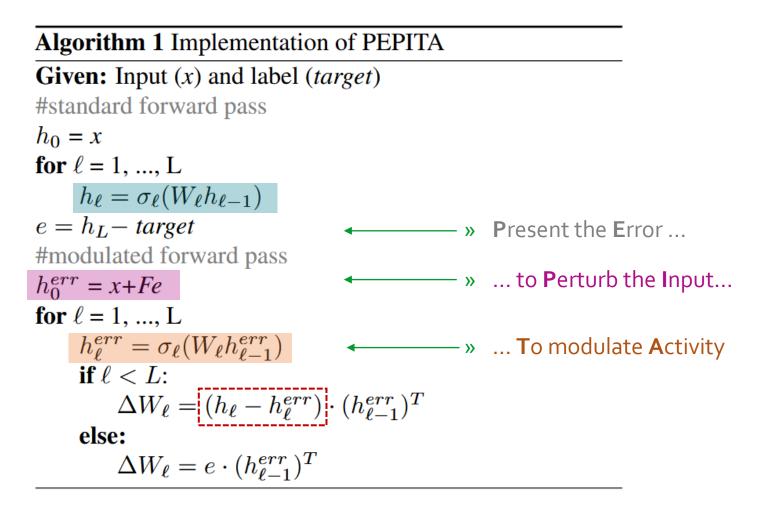
The PEPITA learning rule

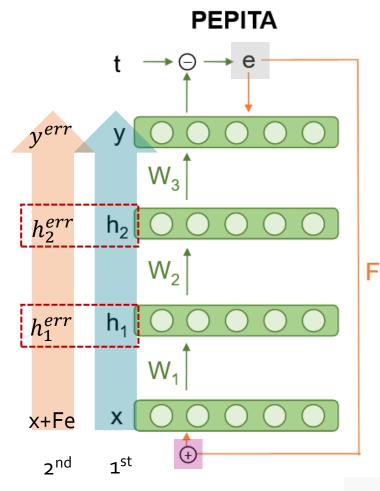
- » **PEPITA** = Present the Error to Perturb the Input To modulate Activity
- » Substitutes the standard Forward+Backward scheme with two Forward Passes
 - <u>Standard</u> Forward pass \rightarrow same as for standard algorithms
 - <u>Modulated</u> Forward pass \rightarrow input is modulated by the error
- » F = projection matrix to add the error onto the input
- » Update relies on difference of activations between Standard and Modulated pass



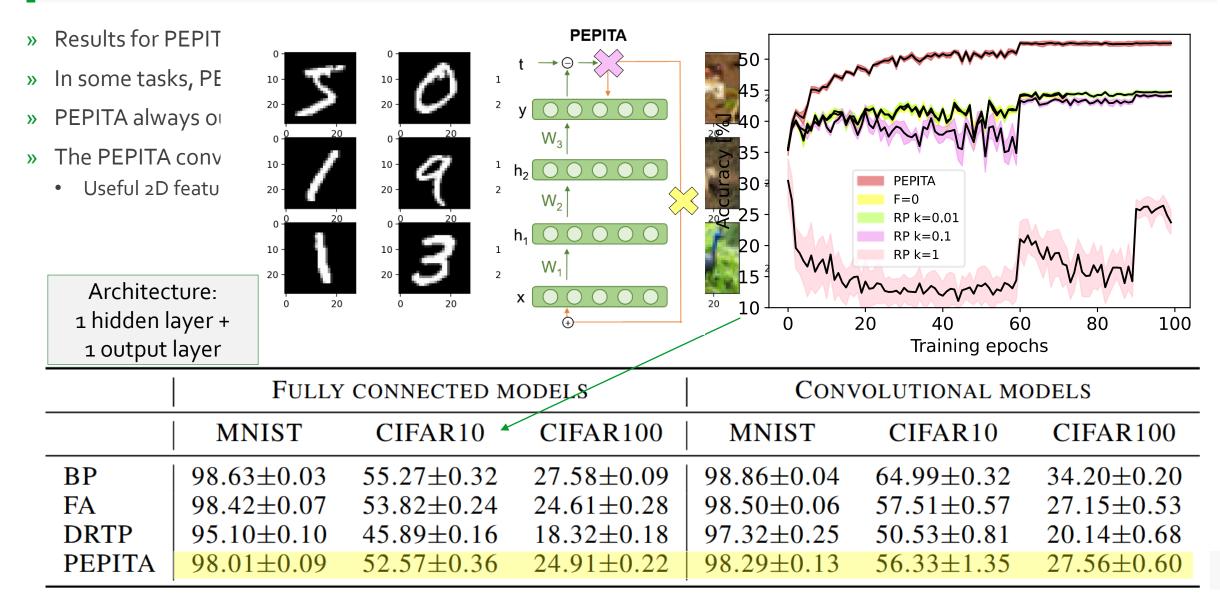
The PEPITA learning rule for Fully Connected Neural Networks

» PEPITA = Present the Error to Perturb the Input To modulate Activity



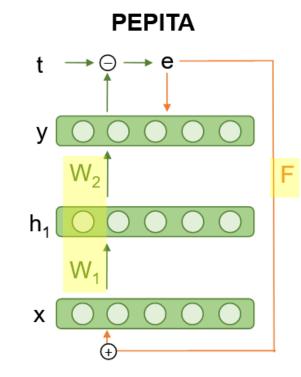


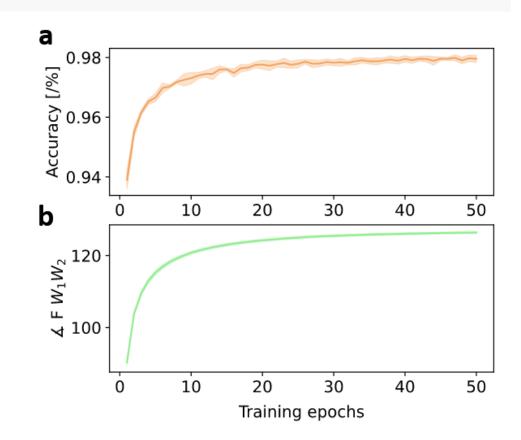
Testing PEPITA on image classification tasks - experimental results



Why it works: soft-antialignment

- » Soft-antialignment
 - Angle between
 - projection matrix F and
 - product between the forward weight matrices
 - Evolution during learning \rightarrow soft antialignment
 - <u>Analytically proven</u> for one-hidden layer linear network





Approximating PEPITA to an Adaptive Feedback Alignment algorithm

» First order Taylor expansion $\Delta W_{\ell} = (h_{\ell} - h_{\ell}^{err}) \cdot (h_{\ell-1}^{err})^{T}$ $f(a+h) \simeq f(a) + hf'(a)$

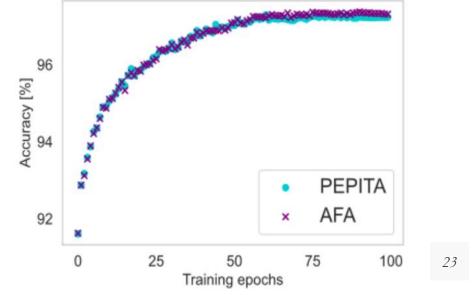
$$h_{1} - h_{1}^{err} = \sigma_{1}(W_{1}x) - \sigma_{1}(W_{1}(x - Fe)) =$$

$$= \sigma_{1}(W_{1}x) - \sigma_{1}(W_{1}x - W_{1}Fe)) =$$

$$\simeq \sigma_{1}(W_{1}x) - [\sigma_{1}(W_{1}(x)) - W_{1}Fe\sigma'_{1}(W_{1}x)] =$$

$$= W_{1}Fe\sigma'_{1}(W_{1}x) =$$

$$= W_{1}Feh'_{1}.$$
FA
$$= W_{1}Feh'_{1}.$$
FA
$$= Adaptive (W) Feedback Alignment$$



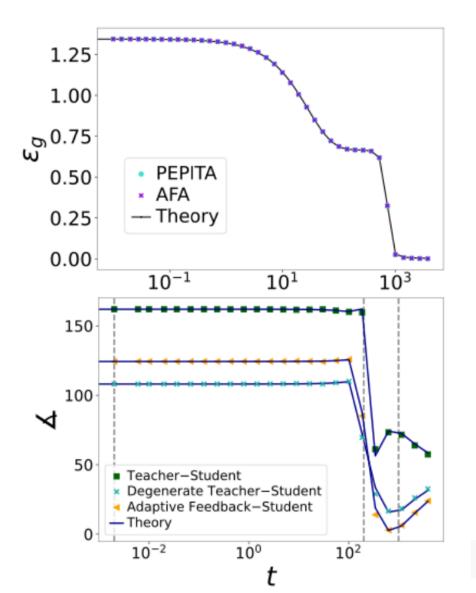
MNIST

FA

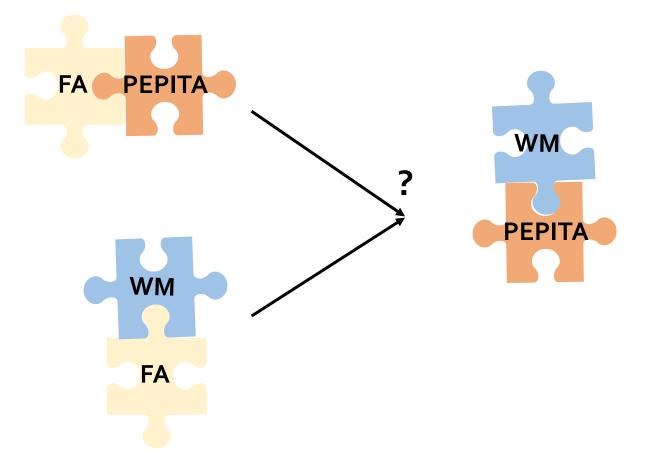
PEPITA

Approximating PEPITA to an Adaptive Feedback Alignment algorithm

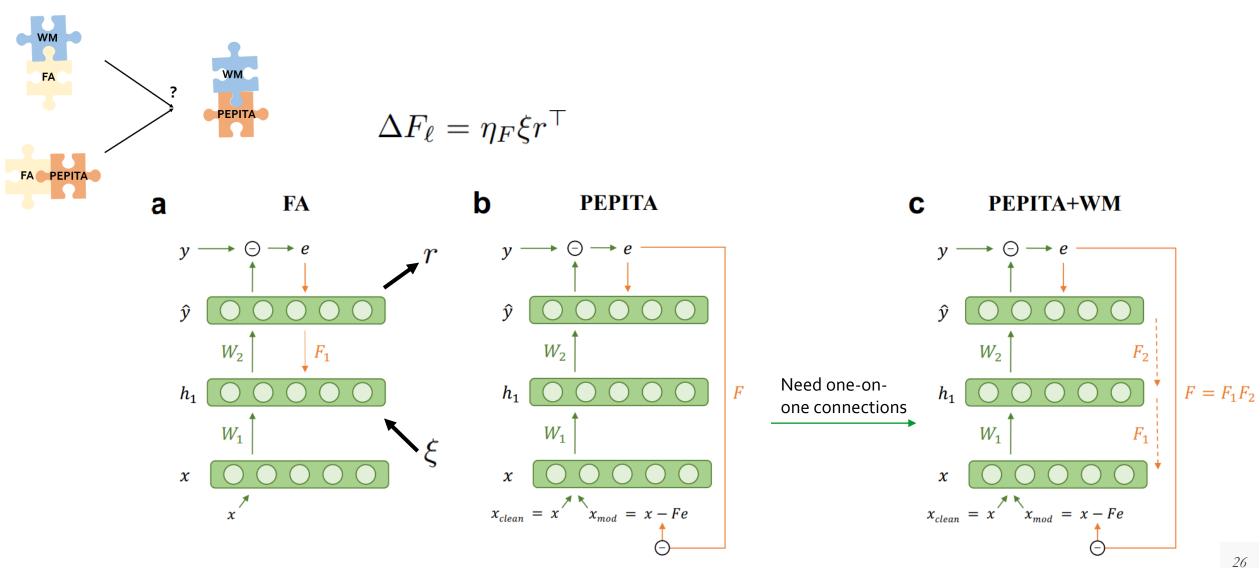
- » Analytical characterization of AFA
 - Teacher student set-up
 - generative model for the data
 - D-dimensional standard Gaussian input vectors
 - label generated by a two-layer teacher network with fixed random weights
 - Two-layer student network trained with AFA and an online (or one-pass)
 - Characterize the dynamics of the mean-squared generalization error
 - infinite-dimensional limit of input dimension and number of samples
 - excellent agreement between infinite-dimensional theory and experiments
 - Dynamics of alignment of the second-layer student weights W2 with:
 - the AF matrix W1F,
 - the second-layer teacher weights,
 - the second-layer teacher weights of the closest degenerate solution.



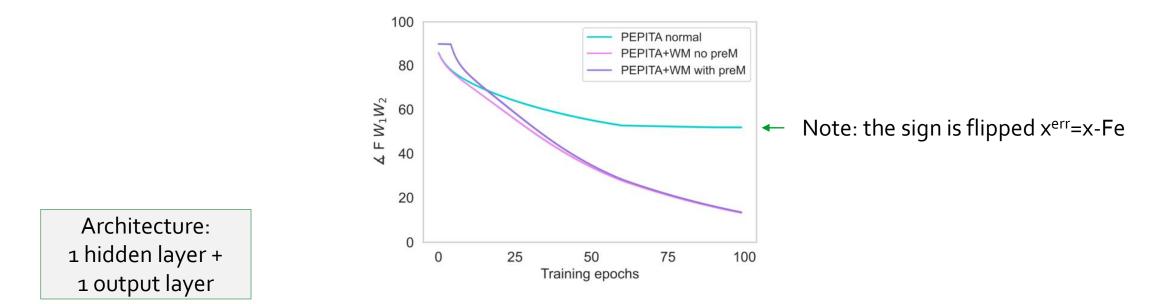
Improving PEPITA's alignment with weight mirroring



Improving PEPITA's alignment with weight mirroring



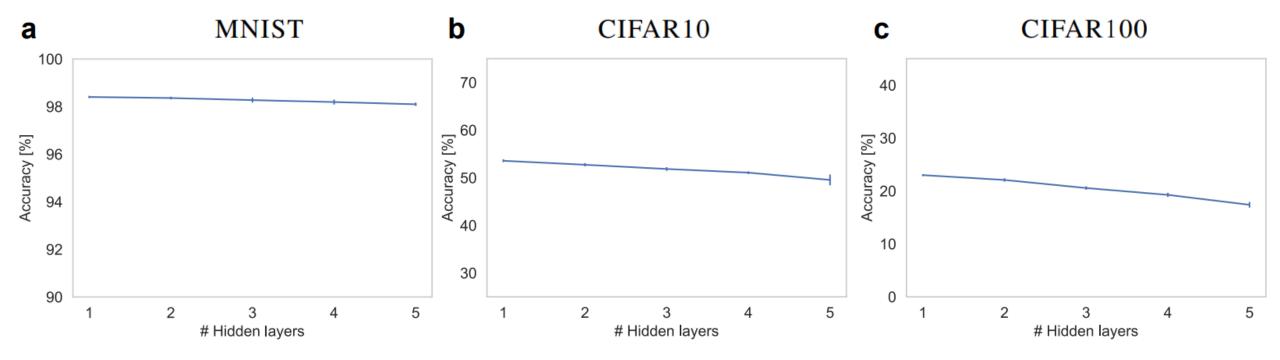
Improving PEPITA's alignment with weight mirroring



	W. DECAY	NORM.	MIRROR	MNIST	CIFAR10	CIFAR100
PEPITA	×	×	×	$98.02{\pm}0.08$	$52.45 {\pm} 0.25$	$24.69 {\pm} 0.17$
	<i>.</i>	X	X	98.12±0.08 98.41±0.08	53.05 ± 0.23 53.51 ± 0.23	24.86 ± 0.18 22.87 ± 0.25
	x	×	2	98.05 ± 0.08	53.51 ± 0.23 52.63 ± 0.30	22.87±0.23 27.07±0.11
	1	×	1	98.10±0.12	$53.46{\pm}0.26$	27.04 ±0.19
	×	√	√	98.42 ±0.05	53.80 ±0.25	24.20 ± 0.36

Training deeper fully connected models

» Adding activation normalization allows to train up to 6 layer networks

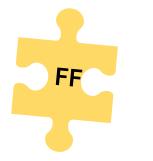


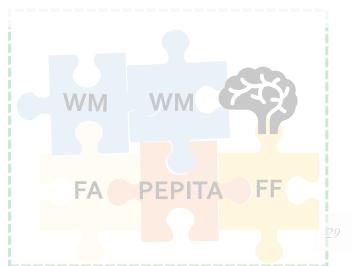
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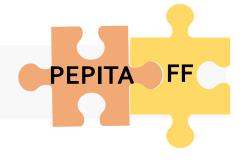




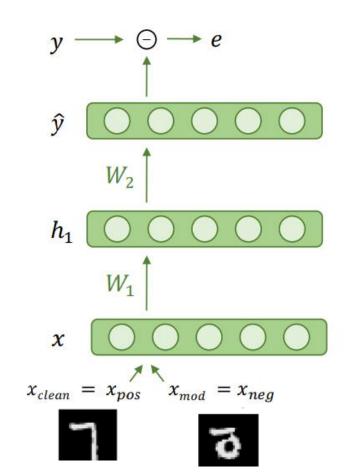


The Forward-Forward algorithm

- » Two forward passes per sample:
 - the positive pass operate on real data
 - the negative pass operates on "negative data"
- » In the positive pass:
 - weights updated to increase the goodness in hidden layers
- » In the negative pass:
 - weights updated to decrease the goodness in hidden layers
- » One measure of goodness
 - sum of the squared neural activities

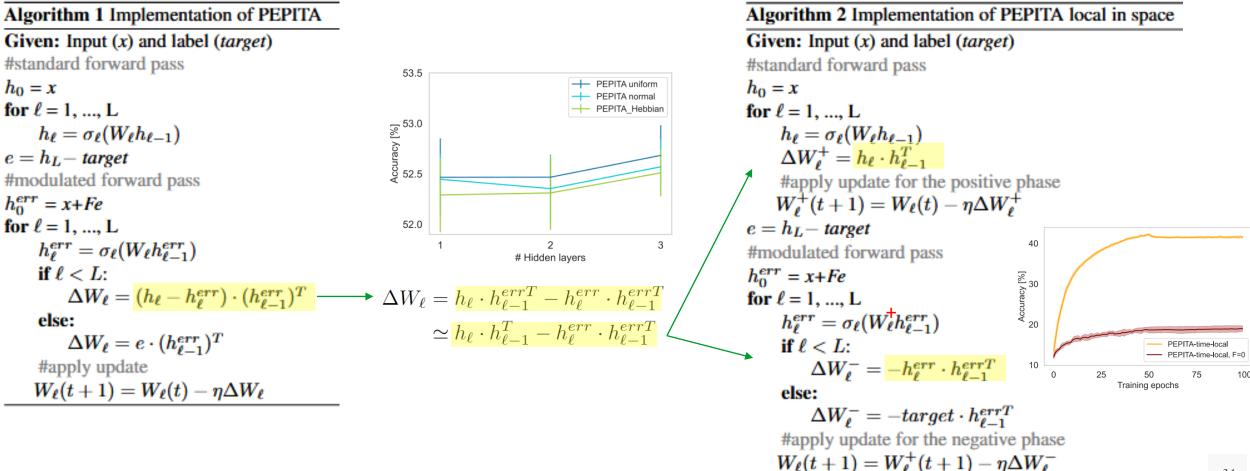


Forward-Forward



Beyond PEPITA: towards time locality

PEPITA



PEPITA 2.0

PEPITA: weight update equivalent to the Forward-Forward framework

Forward-Forward framework

- » Goodness as the sum of squared neural activities
 - h_{l}^{2} for the positive pass and
 - (h^{err})² for the negative pass.
- » Local loss function J_I for layer I = the sum of
 - loss function of the positive pass $\mathsf{J^+_l}$ and
 - loss function of the negative pass J_{I}^{-}

 $J_{\ell} = \|h_{\ell}\|^2 - \|h_{\ell}^{err}\|^2$

PEPITA- Hebbian

$$\Delta W_{\ell} = h_{\ell} \cdot h_{\ell-1}^{errT} - h_{\ell}^{err} \cdot h_{\ell-1}^{errT}$$
$$\simeq \frac{h_{\ell} \cdot h_{\ell-1}^{T} - h_{\ell}^{err} \cdot h_{\ell-1}^{errT}}{h_{\ell}^{errT} - h_{\ell}^{errT} \cdot h_{\ell-1}^{errT}}$$

Equivalence of weight update

$$\frac{1}{2}\frac{\partial J_{\ell}}{\partial W_{\ell}} = \frac{1}{2}\left(\frac{\partial \|h_{\ell}\|^2}{\partial W_{\ell}} - \frac{\partial \|h_{\ell}^{err}\|^2}{\partial W_{\ell}}\right)$$
$$= \frac{1}{2}\left(\frac{\partial \|\sigma(W_{\ell}h_{\ell-1})\|^2}{\partial W_{\ell}} - \frac{\partial \|\sigma(W_{\ell}h_{\ell-1}^{err})\|^2}{\partial W_{\ell}}\right)$$

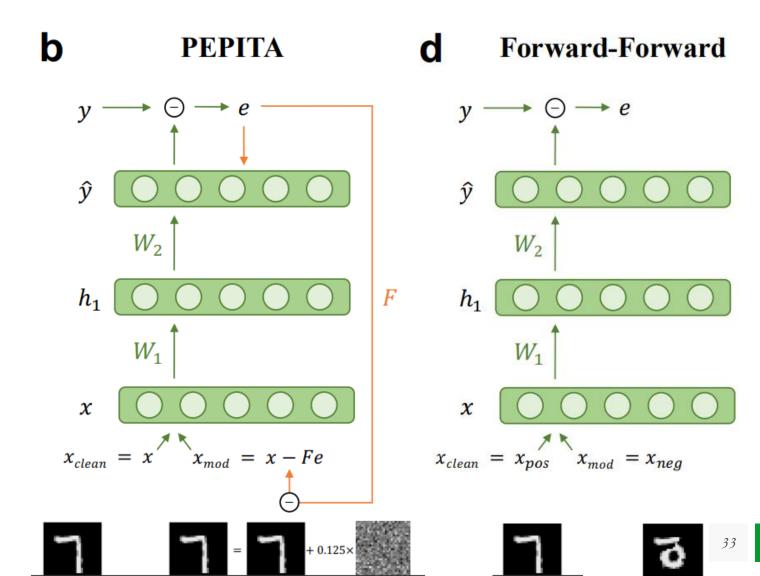
$$= \sigma(W_{\ell}h_{\ell-1}) \odot \sigma'(W_{\ell}h_{\ell-1})h_{\ell-1}^{\top} - \sigma(W_{\ell}h_{\ell-1}^{err}) \odot \sigma'(W_{\ell}h_{\ell-1}^{err})h_{\ell-1}^{err\top} = (\sigma'(W_{\ell}h_{\ell-1}) \odot h_{\ell}h_{\ell-1}^{\top} - \sigma'(W_{\ell}h_{\ell-1}^{err}) \odot h_{\ell}^{err}h_{\ell-1}^{err}) = (h_{\ell}' \odot h_{\ell})h_{\ell-1}^{\top} - (h_{\ell}^{err'} \odot h_{\ell}^{err})h_{\ell-1}^{err\top}.$$
(9)

If ReLU non-linearity:

$$\frac{1}{2}\frac{\partial J_{\ell}}{\partial W_{\ell}} = h_{\ell}h_{\ell-1}^{\top} - h_{\ell}^{err}h_{\ell-1}^{err}^{\top}$$

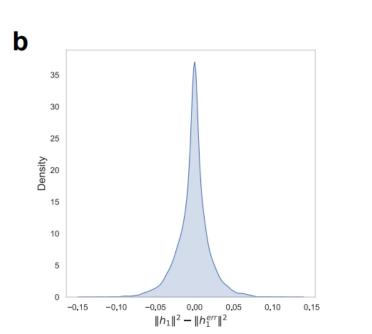
Differences between PEPITA and FF

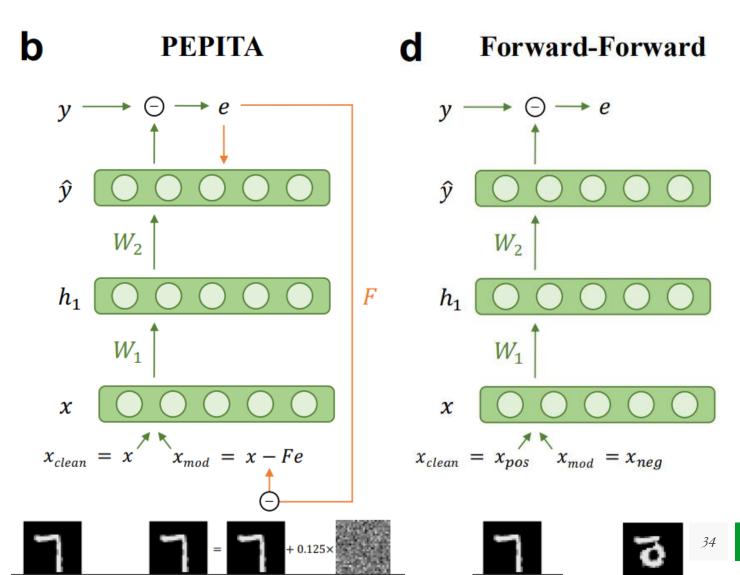
- » Method to generate the modulated sample
 - PEPITA \rightarrow add error \rightarrow top-down feedback
 - FF \rightarrow hybrid mask



Differences between PEPITA and FF

- » Method to generate the modulated sample
 - PEPITA \rightarrow add error \rightarrow top-down feedback
 - FF \rightarrow hybrid mask
- PEPITA does not maximize (resp. minimize) the activations in the clean (resp. modulated) pass





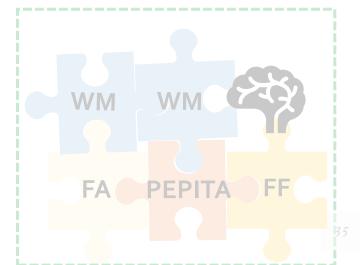
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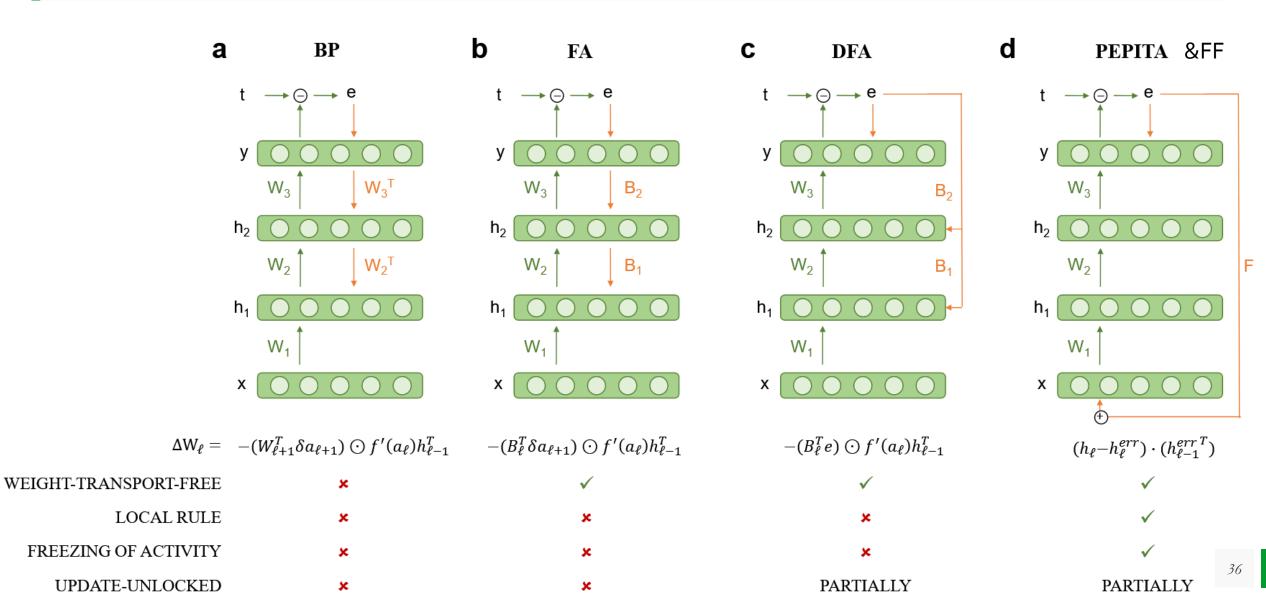








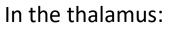
PEPITA solves the biologically implausible aspects of BP



Analysis of PEPITA from a biological standpoint

PEPITA solves the BP's issues of biological plausibility, but it introduces additional elements:

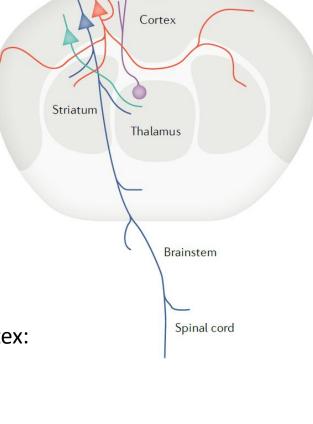
- » Projection of the error onto the input through a fixed random matrix
 - Reminiscent of cortico-thalamo-cortical loops



- Thalamocortical (TC) neurons

Excitatory neurons in the neocortex:

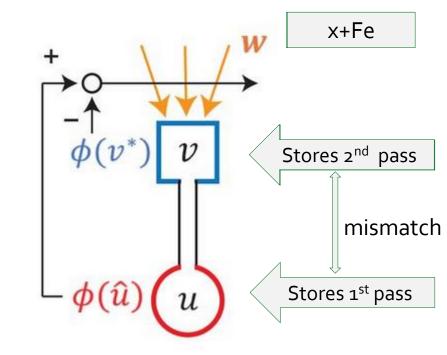
- Intratelencephalic (IT)
- Pyramidal tract (PT)
- Corticothalamic (CT) neurons



Analysis of PEPITA from a biological standpoint

PEPITA solves the BP's issues of biological plausibility, but it introduces additional elements:

- » Projection of the error onto the input through a fixed random matrix
 - Reminiscent of cortico-thalamo-cortical loops
- » Storing of the activation of the *Standard pass* until the *Modulated pass*
 - Can be implemented in biological neurons through mismatch between dendritic and somatic activity



Summary and Outlook

» PEPITA and FF

- Are novel training schemes relying only on **forward computations**
- Solve weight transport, freezing of neural activity, non-local weight updates and backward locking
- Achieve performance on-par with FA on simple image classification tasks
- PEPITA's weight update is that of FF with top-down feedback

» PEPITA

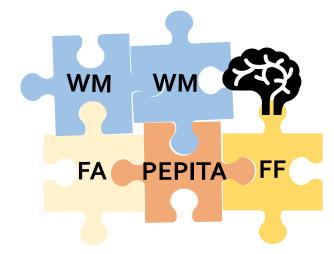
- Can be approximated to Adaptive Feedback Alignment to characterize its dynamics
- Its performance benefits from better alignment (weight mirroring)

» Challenges

- Performance does not improve with depth
- Residual connection, intermediate error-driven modulation, training the F matrix

» Promising avenues for exploration

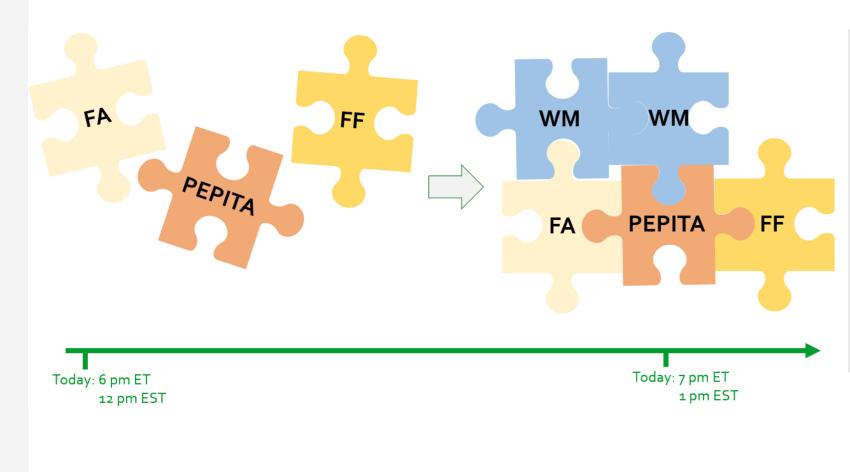
- PEPITA is not gradient-based: could it be more robust to gradient-based adversarial attacks?
- Application to object recognition on videos:
 - Consecutive frames need only one forward pass



Thank. you for your attention!

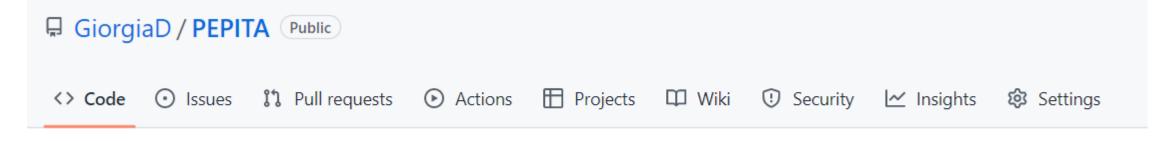
- » Questions?
- » Ideas?
- » Suggestions?

🔀 giorgia.dellaferrera@gmail.com



Coding tutorial: Implementing PEPITA with Pytorch 1/11

- » Today → Code (ICML 2022): <u>https://github.com/GiorgiaD/PEPITA</u>
- » Code with Pytorch lightning (arXiv:2302.05440): <u>https://drive.google.com/drive/u/1/folders/1wqHqtZx2NVuxpdjQuYUVVf1A8v-88oFS</u>



 PEPITA / Tutorial_PEPITA_FullyConnectedNets_CIFAR-10.ipynb

Coding tutorial: Implementing PEPITA with Pytorch 1/11

Import libraries

In [1]: # import torch libraries

import torch
import torchvision
import torchvision.transforms as transforms
import torch.nn as nn
import torch.nn.functional as F
import torch.optim as optim
from torch.autograd import Variable

import other libraries

import numpy as np
import matplotlib.pyplot as plt
import copy

Coding tutorial: Implementing PEPITA with Pytorch 2/11

Define Network architecture

```
In [2]: # models with Dropout
        class NetFC1x1024D0cust(nn.Module):
            def init (self):
                super(). init ()
                self.fc1 = nn.Linear(32*32*3,1024,bias=False)
                self.fc2 = nn.Linear(1024, 10, bias=False)
                # initialize the layers using the He uniform initialization scheme
                fc1 nin = 32*32*3 # Note: if dataset is MNIST --> fc1 nin = 28*28*1
                fc1 limit = np.sqrt(6.0 / fc1 nin)
                torch.nn.init.uniform (self.fc1.weight, a=-fc1 limit, b=fc1 limit)
                fc2 nin = 1024
                fc2 \ limit = np.sqrt(6.0 / fc2 nin)
                torch.nn.init.uniform (self.fc2.weight, a=-fc2 limit, b=fc2 limit)
            def forward(self, x, do masks):
                x = F.relu(self.fc1(x))
                # apply dropout --> we use a custom dropout implementation because we nee
                if do masks is not None:
                    x = x * do masks[0]
                x = F.softmax(self.fc2(x))
                return x
```

Coding tutorial: Implementing PEPITA with Pytorch 3/11

Set hyperparameters and train+test the model

```
In [3]: # set hyperparameters
        ## learning rate
        eta = 0.01
        eta decay = 0.1
        eta decay epochs = [60,90]
        ## number of epochs
        num epochs = 3
        ## dropout keep rate
        keep rate = 0.9
        ## loss --> used to monitor performance, but not for parameter updates (PEPITA de
        criterion = nn.CrossEntropyLoss()
        ## optimizer (choose 'SGD' o 'mom')
        optim = 'mom' # --> default in the paper
        if optim == 'SGD':
            gamma = 0
        elif optim == 'mom':
            gamma = 0.9
        ## batch size
        batch size = 64 # --> default in the paper
```

Coding tutorial: Implementing PEPITA with Pytorch 4/11

```
In [4]: # initialize the network
    net = NetFC1x1024DOcust()
```

In [5]: # define B --> this is the F projection matrix in the paper (here named B because nin = 32*32*3 sd = np.sqrt(6/nin) B = (torch.rand(nin,10)*2*sd-sd)*0.05 # B is initialized with the He uniform in

```
In [6]: # load the dataset - CIFAR-10
```

Files already downloaded and verified Files already downloaded and verified

Coding tutorial: Implementing PEPITA with Pytorch 5/11

```
In [7]: # define function to register the activations --> we need this to compare the activation = {}
    def get_activation(name):
        def hook(model, input, output):
            activation[name] = output.detach()
        return hook
for name, layer in net.named_modules():
        layer.register_forward_hook(get_activation(name))
```

```
In [8]: # do one forward pass to get the activation size needed for setting up the dropod
dataiter = iter(trainloader)
images, labels = next(dataiter)
images = torch.flatten(images, 1) # flatten all dimensions except batch
outputs = net(images,do_masks=None)
layers_act = []
for key in activation:
    if 'fc' in key or 'conv' in key:
        layers_act.append(F.relu(activation[key]))
```

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```
In [9]: # set up for momentum
         if optim == 'mom':
             gamma = 0.9
             v w all = []
             for l idx,w in enumerate(net.parameters()):
                 if len(w.shape)>1:
                     with torch.no grad():
                         v w all.append(torch.zeros(w.shape))
In [10]: # Train and test the model
         test accs = []
         for epoch in range(num epochs): # loop over the dataset multiple times
             # learning rate decay
             if epoch in eta decay epochs:
                 eta = eta*eta decay
                 print('eta decreased to ',eta)
             # Loop over batches
             running loss = 0.0
             for i, data in enumerate(trainloader, 0):
                 # get the inputs; data is a list of [inputs, labels]
                 inputs, target = data
                 inputs = torch.flatten(inputs, 1) # flatten all dimensions except batch
                 target onehot = F.one hot(target,num classes=10)
```

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```
# create dropout mask for the two forward passes --> we need to use the
do masks = []
if keep rate < 1:
    for l in layers act[:-1]:
        input1 = 1
        do mask = Variable(torch.ones(inputs.shape[0],input1.data.new(input)
        do masks.append(do mask)
    do masks.append(1) # for the last layer we don't use dropout --> just
# forward pass 1 with original input --> keep track of activations
outputs = net(inputs,do masks)
layers act = []
cnt act = 0
for key in activation:
    if 'fc' in key or 'conv' in key:
        layers act.append(F.relu(activation[key])* do masks[cnt act]) #
        cnt act += 1
# compute the error
error = outputs - target onehot
# modify the input with the error
error input = error @ B.T
mod inputs = inputs + error input
```

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```
# forward pass 2 with modified input --> keep track of modulated activaty
mod_outputs = net(mod_inputs,do_masks)
mod_layers_act = [] .
cnt_act = 0
for key in activation:
    if 'fc' in key or 'conv' in key:
        mod_layers_act.append(F.relu(activation[key])* do_masks[cnt_act]]
        cnt_act += 1
mod_error = mod_outputs - target_onehot
```

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```
# compute the delta w for the batch
delta w all = []
v w = []
for l idx,w in enumerate(net.parameters()):
    v w.append(torch.zeros(w.shape))
for l in range(len(layers act)):
    # update for the last layer
    if l == len(layers act)-1:
        if len(layers act)>1: # if network has more than one layer
            delta w = -mod error.T @ mod layers act[-2]
        else: # if only one layer network
            delta w = -mod error.T @ mod inputs
    # update for the first layer
    elif ] == 0:
        delta w = -(layers act[1] - mod layers act[1]).T @ mod inputs
    # update for the hidden layers (not first, not last)
    elif l>0 and l<len(layers act)-1:</pre>
        delta w = -(layers act[1] - mod layers act[1]).T @ mod layers act
    delta w all.append(delta w)
```

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```
# apply the weight change
if optim == 'SGD': # if SGD without momentum
    for l idx,w in enumerate(net.parameters()):
        with torch.no grad():
            w += eta * delta w all[1 idx]/batch size # specify for which
elif optim == 'mom': # if SGD with momentum
    for l idx,w in enumerate(net.parameters()):
        with torch.no grad():
            v w all[l idx] = gamma * v w all[l idx] + eta * delta w all[]
            w += v w all[1 idx]
# keep track of the loss
loss = criterion(outputs, target)
# print statistics
running loss += loss.item()
if i%500 == 499:
    print('[%d, %5d] loss: %.3f' %
          (epoch + 1, i + 1, running loss / 500))
    running loss = 0.0
```

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```
print('Testing...')
   correct = 0
   total = 0
   # since we're not training, we don't need to calculate the gradients for our
    with torch.no grad():
        for test data in testloader:
            test images, test labels = test data
            test images = torch.flatten(test images, 1) # flatten all dimensions
            # calculate outputs by running images through the network
            test outputs = net(test images,do masks=None)
            # the class with the highest energy is what we choose as prediction
            _, predicted = torch.max(test_outputs.data, 1)
            total += test labels.size(0)
            correct += (predicted == test labels).sum().item()
    print('Test accuracy epoch {}: {} %'.format(epoch, 100 * correct / total))
    test accs.append(100 * correct / total)
print('Finished Training')
```